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Complex Intuitionistic Fuzzy SuperHypergraph and Complex q-Rung Orthopair Fuzzy SuperHypergraph with Illustrative Real-World Examples

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Abstract: Classical graph theory models a system by representing entities as vertices and pairwise relationships as edges. Hypergraphs extend this paradigm by allowing a *hyperedge* to connect any nonempty subset of vertices, thereby capturing multiway interactions in a single combinatorial object. Superhypergraphs push this idea further by iterating the powerset construction, which enables one to encode nested, hierarchical, and even self-referential incidence patterns across multiple levels within a unified framework. Over the past decades, graph- and hypergraph-based models have been combined with many uncertainty formalisms (e.g., fuzzy, intuitionistic fuzzy, neutrosophic, hesitant, and plithogenic settings) to better represent vagueness, hesitation, and incomplete information. In this paper, we introduce two superhypergraph-based frameworks that incorporate complex-valued uncertainty information: *Complex Intuitionistic Fuzzy Superhypergraphs* and *Complex q-Rung Orthopair Fuzzy Superhypergraphs*. These models lift the existing complex intuitionistic fuzzy and complex q-rung orthopair fuzzy *hypergraph* constructions to the superhypergraph level, providing a coherent mechanism for modeling uncertainty simultaneously across multiple hierarchical layers. We also establish basic structural properties, clarify their reduction to standard (lower-order) models, and present constructive procedures that support algorithmic use in hierarchical decision and network modeling.

Keywords: Superhypergraph, Complex q-Rung Orthopair Fuzzy SuperHyperGraph, Complex Fuzzy SuperHyperGraph, Fuzzy Superhypergraph, Fuzzy Graph

2010 Mathematics Subject Classification. 06C65.

1 Introduction

1.1 Hypergraphs and SuperHyperGraphs Theory with Applications

Classical networked systems are most commonly modeled by a *graph*, where vertices represent entities and edges encode pairwise connections between them [1]. To describe interactions among arbitrary groups rather than just pairs, the notion of a *hypergraph* was introduced, in which a single hyperedge may link any nonempty subset of the vertex set [2].

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Despite their central role and the extensive literature on both graphs and hypergraphs, these structures are not fully adequate for capturing deeper hierarchical organization or multi-level patterns of connectivity. To address this limitation, we introduce the *Super-HyperGraph*, obtained by iterating the powerset construction so as to generate nested layers of vertices and edges that reflect hierarchical relationships and multi-scale interactions [3]. Graphs and hypergraphs are already widely used in artificial intelligence, network science, neural networks, decision science, control theory, operations research, data mining, chemistry, and physics [4]. For example, in operations research, graphs and hypergraphs model networks, constraints, and multiway interactions, enabling optimization of flows, schedules, assignments, and decisions via shortest paths, matchings, cuts, and higher-order incidence structures (e.g.[5,6,7]). By explicitly incorporating several levels of higher-order connections, Super-HyperGraphs provide a convenient modeling tool for complex systems whose structure spans multiple scales.

Concept	Notation	Edge Domain	Construction Principle
Graph	$G=(V,E)$	$E \subseteq \{\{u,v\} \mid u,v \in V,\ u \neq v\}$	Edges connect exactly two distinct vertices.
Hypergraph	$H=(V,E)$	$E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$	Each hyperedge may cover any nonempty subset of vertices.
Super-HyperGraph	$\mathrm{SuHyG}^{(n)}=(V_0,V,E)$	$V \subseteq \mathrm{POS}^n(V_0),\ E \subseteq \mathrm{POS}(V)$	Iterated powersets (n levels) generate nested connectivity layers.

Table 1: Comparison of graph-based models and their edge/hyperedge domains

Because the Super-HyperGraph framework has substantial explanatory power, it has naturally been extended to uncertainty-aware variants such as Fuzzy Super-HyperGraphs and Neutrosophic Super-HyperGraphs, which in turn have stimulated a variety of subsequent theoretical developments and applications (e.g.,[8,9,10]). From this perspective, the authors believe that research on the Super-HyperGraph framework studied in this paper is likely to become increasingly active in fields such as decision science and operations research.

1.2 Complex Intuitionistic Fuzzy Graph and Related Graph-Based Operational Models

In many practical settings, relationships between entities are affected by vagueness, hesitation, or incomplete information. To describe such phenomena, a rich collection of uncertainty-valued graph models has been put forward, including:

–*Fuzzy graphs*, where each vertex and edge is assigned a single membership degree in the unit interval $[0,1]$[11].
–*Intuitionistic fuzzy graphs*, which in addition specify a non-membership degree in the sense of Atanassov, subject to the usual intuitionistic constraints[12].
–*Neutrosophic graphs*, in which every vertex and edge is characterized by a triplet of truth, indeterminacy, and falsity degrees[13].
–*Plithogenic graphs*, designed to incorporate attribute-wise contradictions on top of fuzzy uncertainty[14].

Collectively, these models support a detailed encoding of uncertainty on vertices and edges, allowing for the explicit representation of vagueness, inconsistency, and indeterminacy, and thereby improving the robustness and expressive power of graph-based decision-support systems. Moreover, concepts such as fuzzy graphs, intuitionistic fuzzy graphs, neutrosophic graphs, and plithogenic graphs have also been widely used in operations research and optimization, as well as in a variety of practical, real-world operational applications[15,16].

In this work, we focus on *Complex Intuitionistic Fuzzy Graphs*[17] and *Complex q-Rung Orthopair Fuzzy Graphs* [18] in the ambient framework of Super-HyperGraphs. In a complex intuitionistic fuzzy graph, each vertex and each edge is equipped with a complex-valued intuitionistic fuzzy pair consisting of a membership magnitude and a non-membership magnitude in $[0,1]$, together with corresponding phase angles, such that the sum of the membership and non-membership magnitudes does not exceed 1. As related concepts to complex intuitionistic fuzzy graphs, Complex Neutrosophic Graphs and similar frameworks have been studied [19,20]. A complex q-rung orthopair fuzzy graph assigns to every vertex

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and edge a complex membership magnitude and a complex non-membership magnitude (again in $[0,1]$, with associated phases) that satisfy the q-rung orthopair condition

$$T^q + F^q \leq 1$$

for each graph element. The q-rung orthopair fuzzy graph thus extends and refines earlier fuzzy and intuitionistic fuzzy graph structures, and in particular contains Pythagorean fuzzy graphs as a special case[21]. Moreover, given the importance of research on q-rung orthopair fuzzy graphs, related concepts such as Pythagorean neutrosophic graphs [22, 23] have also been introduced.

Moreover, hypergraph counterparts of these models, namely *Complex Intuitionistic Fuzzy Hypergraphs* [24] and *Complex q-Rung Orthopair Fuzzy Hypergraphs* [24], have also been formulated, and their potential in domains such as decision science and engineering has been actively investigated. For reference, a comparison of IFHG, CIFHG, and Cq-ROFHG is presented in Table 2.

Table 2: Concise comparison of IFHG, CIFHG, and Cq-ROFHG

Aspect	IFHG	CIFHG	Cq-ROFHG
Underlying structure	Hypergraph (V,E)	Hypergraph (V,E)	Hypergraph (V,E)
Uncertainty data on vertices/edges	Real pair $(T,F) \in [0,1]^2$	Complex pair $\left(Te^{i\varphi}, Fe^{i\psi}\right)$ with $T,F \in [0,1]$	Complex pair $\left(Te^{i\varphi}, Fe^{i\psi}\right)$ with $T,F \in [0,1]$
Primary constraint (node/edgewise)	$T+F \leq 1$	$T+F \leq 1$	$T^q+F^q \leq 1$ (fixed $q \geq 1$)
Role of phase angles	None (real-valued)	Phases $\varphi,\psi \in [0,2\pi]$ encode additional (directional/periodic) info	Phases $\varphi,\psi \in [0,2\pi]$ encode additional (directional/periodic) info
Relation among families	Base (real) intuitionistic model	Complex-valued extension of IFHG (set phases 0 to recover IFHG-like data)	Generalizes orthopair models; for $q=1$ reduces to complex IF-type constraint
Special cases	Fuzzy HG by setting $F = 1-T$ or by discarding F	Real IFHG if $\varphi=\psi=0$	Complex IF-type model when $q=1$; more permissive for larger q

1.3 Our Contribution

While *Complex Intuitionistic Fuzzy Hypergraphs* and *Complex q-Rung Orthopair Fuzzy Hypergraphs* have already been proposed in the literature, their extensions to the Super-HyperGraph setting have, to the best of our knowledge, not yet been systematically investigated. As emphasized above, Super-HyperGraphs provide a natural framework for capturing hierarchical and multi-level relationships.

To bridge this gap, we propose two novel frameworks: *Complex Intuitionistic Fuzzy Super-HyperGraphs* and *Complex q-Rung Orthopair Fuzzy Super-HyperGraphs*. These frameworks embed the existing hypergraph models into the Super-HyperGraph paradigm, enabling multi-level uncertainty modeling within hierarchical graph structures. Moreover, we believe that *Complex Intuitionistic Fuzzy Super-HyperGraphs* and *Complex q-Rung Orthopair Fuzzy Super-HyperGraphs* will make it easier to conduct research in Operations Research on complex concepts and models. For reference, Table 3 provides a concise comparison of Complex Intuitionistic Fuzzy HyperGraphs and their Super-HyperGraph counterparts, and Table 4 summarizes the corresponding comparison for Complex q-Rung Orthopair Fuzzy HyperGraphs versus Complex q-Rung Orthopair Fuzzy Super-HyperGraphs.

1.4 Structure of this paper

The organization of this paper is outlined as follows. Section 2 reviews the necessary preliminaries on Hypergraphs and SuperHyperGraphs, together with the notions of Complex Intuitionistic Fuzzy HyperGraphs and Complex q-Rung

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Table 3: Complex Intuitionistic Fuzzy HyperGraphs vs. Super-HyperGraphs

Aspect	Complex Intuitionistic Fuzzy HyperGraphs	Complex Intuitionistic Fuzzy Super-HyperGraphs
Underlying objects	Vertices and hyperedges on a base set S	n-supervertices and n-superedges on $\mathrm{POS}^n(S)$
Hierarchy	Single-level (flat) structure	Explicit multi-level (hierarchical) structure
Uncertainty on nodes/edges	Complex intuitionistic membership and non-membership	Same complex intuitionistic data on supervertices and superedges
Main focus	Uncertainty in relations among basic entities	Uncertainty in relations among clusters of entities
Reduction relation	Base model	Reduces to the hypergraph case when $n=0$

Table 4: Complex q-Rung Orthopair Fuzzy HyperGraphs vs. Super-HyperGraphs

Aspect	Complex q-Rung Orthopair Fuzzy HyperGraphs	Complex q-Rung Orthopair Fuzzy Super-HyperGraphs
Underlying objects	Vertices and hyperedges on a base set S	n-supervertices and n-superedges on $\mathrm{POS}^n(S)$
Hierarchy	Single-level (non-hierarchical) structure	Explicit multi-level (hierarchical) structure
q-rung constraint	Complex orthopair degrees (T,F) on vertices/edges with $T^q+F^q\leq 1$	Same orthopair constraint on supervertices/superedges at each level
Uncertainty representation	Complex membership/non-membership for basic entities	Complex membership/non-membership for clusters of entities
Modeling focus	Local relations among atomic objects	Relations among aggregated clusters across levels
Reduction relation	Base q-rung orthopair fuzzy hypergraph model	Reduces to the hypergraph case when $n=0$

Orthopair Fuzzy HyperGraphs. Section 3 develops extensions of these two fuzzy–hypergraph models to the SuperHyperGraph setting and investigates their structural properties. Section 4 presents algorithms for the proposed frameworks and examines their correctness and computational characteristics. Section 5 provides concluding remarks and directions for future research.

2 Preliminaries

In this section we introduce the key notions and notation used throughout.

2.1 Hypergraphs and SuperHyperGraphs

In this subsection, we recall the basic definitions of hypergraphs and n-superhypergraphs. Throughout this paper, we fix an integer $n\geq 0$. For each nonnegative integer k, the notation $\mathrm{POS}^k(\cdot)$ denotes the k-fold iteration of the powerset operator on *finite* sets.

Definition 1(Hypergraph). *[25] A* (finite) hypergraph *is an ordered pair $H=(V,E)$ such that*

–*V is a finite set (the set of* vertices*), and*

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–$E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$ *is a finite family of nonempty subsets of* V *(the members of* E *are called* hyperedges*).*

Definition 2(Iterated powerset). *(cf. [26]) Let* X *be a finite set. Define* $\mathrm{POS}^k(X)$ *recursively by*

$$\mathrm{POS}^0(X) := X, \qquad \mathrm{POS}^{k+1}(X) := \mathrm{POS}\big(\mathrm{POS}^k(X)\big), \quad k \geq 0.$$

The set $\mathrm{POS}^n(X)$ *is called the* n-th iterated powerset *of* X.

Definition 3(n-SuperHyperGraph). *[3] Let* V_0 *be a finite nonempty base set and let* $n \geq 0$. *With* $\mathrm{POS}^k(V_0)$ *as in Definition 2, an* n-superhypergraph *on* V_0 *is a triple*

$$\mathrm{SuHyG}^{(n)} = \big(V_0, V^{(n)}, E^{(n)}\big),$$

where

–$V^{(n)} \subseteq \mathrm{POS}^n(V_0)$ *is a finite set of* n-supervertices*, and*
–$E^{(n)} \subseteq \mathrm{POS}\big(V^{(n)}\big) \setminus \{\emptyset\}$ *is a finite family of nonempty subsets of* $V^{(n)}$*, called* n-superedges*.*

Equivalently, each $e \in E^{(n)}$ *satisfies* $e \subseteq V^{(n)}$ *and* $e \neq \emptyset$. *When* V_0 *is clear from context, we write* $\mathrm{SuHyG}^{(n)} = (V^{(n)}, E^{(n)})$. *For* $n = 0$*, this definition reduces to the usual notion of a (finite) hypergraph on* V_0*, since* $\mathrm{POS}^0(V_0) = V_0$*,* $V^{(0)} \subseteq V_0$*, and* $E^{(0)} \subseteq \mathrm{POS}\big(V^{(0)}\big) \setminus \{\emptyset\}$.

Example 1(World map as a 2*-superhypergraph).* Let V_0 be the set of all cities on Earth. For each country k, let $C_k \subseteq V_0$ denote the set of cities in country k, and define

$$V^{(1)} := \{\, C_k \mid k \text{ is a country} \,\} \subseteq \mathrm{POS}(V_0).$$

For each continent c, define the family of country-sets belonging to c by

$$D_c := \{\, C_k \in V^{(1)} \mid \text{country } k \text{ lies in continent } c \,\}.$$

Then $D_c \subseteq V^{(1)}$, hence $D_c \in \mathrm{POS}(V^{(1)}) \subseteq \mathrm{POS}(\mathrm{POS}(V_0)) = \mathrm{POS}^2(V_0)$. Set

$$V^{(2)} := \{\, D_c \mid c \text{ is a continent} \,\} \subseteq \mathrm{POS}^2(V_0).$$

Now group continents into hemispheres by defining

$$H_{\text{Eastern}} := \{\, D_c \in V^{(2)} \mid c \text{ lies in the Eastern Hemisphere} \,\},$$

$$H_{\text{Western}} := \{\, D_c \in V^{(2)} \mid c \text{ lies in the Western Hemisphere} \,\}.$$

Each H_{Eastern} and H_{Western} is a nonempty subset of $V^{(2)}$, so $H_{\text{Eastern}}, H_{\text{Western}} \in \mathrm{POS}(V^{(2)}) \setminus \{\emptyset\}$. Define

$$E^{(2)} := \{\, H_{\text{Eastern}}, H_{\text{Western}} \,\} \subseteq \mathrm{POS}(V^{(2)}) \setminus \{\emptyset\}.$$

Therefore

$$\mathrm{SuHyG}^{(2)} = \big(V_0, V^{(2)}, E^{(2)}\big)$$

is a 2-superhypergraph on V_0. Here the 2-supervertices D_c represent continents (as families of country-sets), and the 2-superedges H_{Eastern} and H_{Western} represent hemispheres (as groupings of continents). For reference, a schematic overview of this example is provided in Figure 1.

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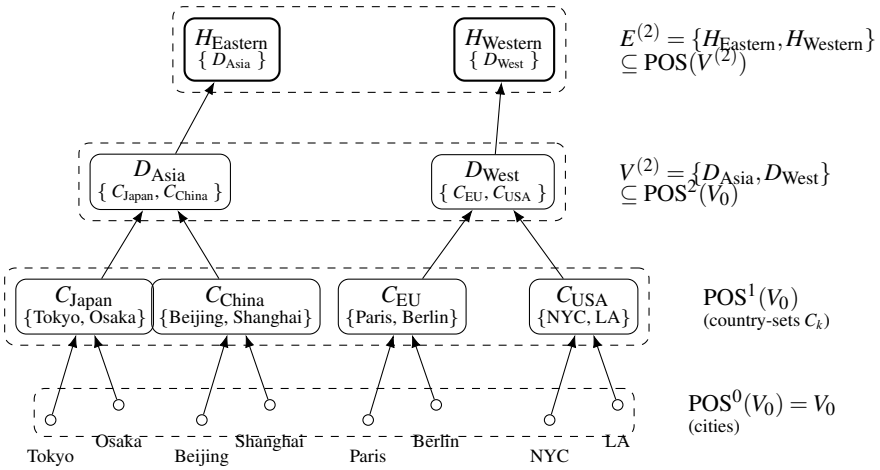

Fig. 1: Schematic illustration of the example.

2.2 Fuzzy n-SuperHyperGraphs

In this subsection, we present the definition of a fuzzy n-superhypergraph. ‘

Definition 4(Fuzzy n-SuperHyperGraph). *[3] Let* $\text{SuHyG}^{(n)} = (V, E)$ *be a (finite) n-SuperHyperGraph. A (finite)* fuzzy n-SuperHyperGraph *on* $\text{SuHyG}^{(n)}$ *is a quadruple*

$$\big(V, E, \sigma, \mu\big),$$

where

- $\sigma : V \to [0,1]$ *is a membership function on the (finite) set of n-supervertices; for each* $v \in V$*, the quantity* $\sigma(v)$ *gives the membership degree of v,*
- $\mu : E \to [0,1]$ *is a membership function on the (finite) set of n-superedges; for each* $e \in E$*, the value* $\mu(e)$ *specifies the membership degree of e.*

The functions σ *and* μ *are required to satisfy the constraint*

$$\mu(e) \;\le\; \min_{v \in e} \sigma(v) \qquad \textit{for all } e \in E,$$

which expresses that the membership degree of a superedge cannot exceed the least membership degree of its incident supervertices.

Example 2(Hierarchical supply–chain structure represented as a finite fuzzy 2*-SuperHyperGraph).* We model a simple distribution network with four warehouses

$$S = \{W_1, W_2, W_3, W_4\}.$$

First, group them into two level-1 warehouse clusters

$$T_1 = \{W_1, W_2\}, \qquad T_2 = \{W_3, W_4\},$$

so that $T_1, T_2 \in \text{POS}^1(S)$. At the next level, introduce the supervertices

$$G_1 = \{T_1\}, \qquad G_2 = \{T_2\},$$

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which belong to $\mathrm{POS}^2(S)$. We take

$$V = \{G_1, G_2\}, \qquad E = \big\{ e = \{G_1, G_2\} \big\}.$$

Assign fuzzy membership grades representing on-time delivery performance:

$$\sigma(G_1) = 0.90, \qquad \sigma(G_2) = 0.85.$$

For the unique superedge, set

$$\mu(e) = 0.80.$$

Here $\mu(e)$ can be interpreted as the *reliability of the aggregated inter-cluster link* (or the *end-to-end on-time coordination level*) between the two warehouse clusters—for instance, the degree to which the joint operation (handoffs, transfers, and shared transportation capacity) between G_1 and G_2 meets the on-time delivery requirement.

Then

$$\mu(e) \;\le\; \min\{\sigma(G_1), \sigma(G_2)\} \;=\; \min\{0.90, 0.85\} \;=\; 0.85,$$

so the edge-membership constraint is satisfied.

Consequently,

$$\big(V, E, \sigma, \mu\big)$$

constitutes a fuzzy 2-SuperHyperGraph that encodes hierarchical warehouse clusters and the overall reliability of their combined link. For reference, a schematic overview of this example is shown in Figure 2.

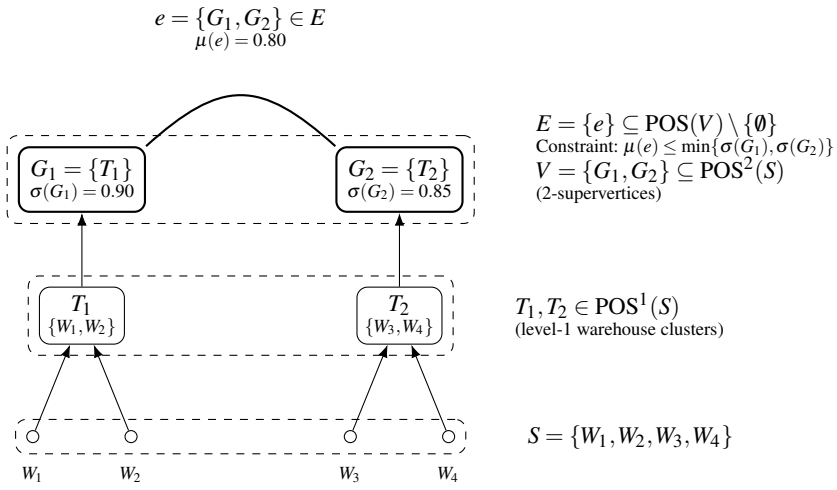

Fig. 2: Hierarchical supply–chain structure as a finite fuzzy 2-superhypergraph: warehouses → clusters → 2-supervertices, with a single superedge e.

2.3 Hypergraph with Complex Intuitionistic Fuzzy Information

In this subsection, we present the definition of a Complex Intuitionistic Fuzzy Hypergraph.

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Definition 5(Complex Intuitionistic Fuzzy Hypergraph). *(cf.[24]) Let Y be a finite nonempty set. A* complex intuitionistic fuzzy hypergraph *(*CIFHG*) on Y is a pair*

$$H = (C, D),$$

where C is a finite family *of complex intuitionistic fuzzy sets on Y (interpreted as complex intuitionistic fuzzy hyperedges) and D assigns to each such hyperedge an aggregated complex intuitionistic label, as follows.*

–Complex intuitionistic fuzzy hyperedges. *Let $I := \{1, \dots, k\}$. The family*

$$C = (\alpha_j)_{j \in I}$$

consists of complex intuitionistic fuzzy sets α_j on Y. For each $j \in I$, α_j is specified by four functions

$$T_{\alpha_j}, F_{\alpha_j} : Y \to [0,1], \qquad \varphi_{\alpha_j}, \psi_{\alpha_j} : Y \to [0, 2\pi],$$

satisfying, for every $u \in Y$,

$$T_{\alpha_j}(u) + F_{\alpha_j}(u) \leq 1.$$

We write the complex membership and non-membership degrees of u (with respect to α_j) as

$$\mu_{\alpha_j}(u) := T_{\alpha_j}(u) e^{i\varphi_{\alpha_j}(u)}, \qquad \nu_{\alpha_j}(u) := F_{\alpha_j}(u) e^{i\psi_{\alpha_j}(u)}.$$

–Underlying crisp hyperedge family (supports). *For each $j \in I$, define the (crisp) support of α_j by*

$$E_j := \operatorname{supp}(\alpha_j) := \left\{ u \in Y : \ T_{\alpha_j}(u) > 0 \text{ or } F_{\alpha_j}(u) > 0 \right\} \subseteq Y.$$

We assume $E_j \neq \emptyset$ for all $j \in I$. The resulting (possibly repeated) crisp hyperedge family is

$$\mathscr{E} := (E_j)_{j \in I}.$$

(Thus multiple hyperedges are allowed: distinct indices may yield the same subset of Y.)

–Edge-level aggregated complex intuitionistic label. *The mapping D is given on the index set I (equivalently, on the edge family $\mathscr{E}$ with multiplicity) by*

$$D : I \to [0,1]^2 \times [0, 2\pi]^2, \qquad D(j) = \big(T_D(j), F_D(j), \varphi_D(j), \psi_D(j)\big),$$

where $T_D(j), F_D(j) \in [0,1]$ and $\varphi_D(j), \psi_D(j) \in [0, 2\pi]$. The corresponding complex intuitionistic pair assigned to the j-th hyperedge is

$$\big(T_D(j) e^{i\varphi_D(j)}, \ F_D(j) e^{i\psi_D(j)}\big).$$

These data must satisfy, for every $j \in I$,

$$T_D(j) + F_D(j) \leq 1, \tag{1}$$

$$T_D(j) \leq \min_{u \in E_j} T_{\alpha_j}(u), \qquad F_D(j) \leq \max_{u \in E_j} F_{\alpha_j}(u), \tag{2}$$

$$\varphi_D(j) \leq \min_{u \in E_j} \varphi_{\alpha_j}(u), \qquad \psi_D(j) \geq \max_{u \in E_j} \psi_{\alpha_j}(u). \tag{3}$$

Finally, the family C is required to cover the vertex set in the sense that every $u \in Y$ belongs to (at least) one hyperedge support:

$$\bigcup_{j \in I} \operatorname{supp}(\alpha_j) = Y.$$

Remark(On phase aggregation in complex fuzzy hypergraphs). In Definitions 2.7 and 2.8, the phase components $\varphi(\cdot), \psi(\cdot) \in [0, 2\pi]$ represent *angular* (cyclic) information. Since there is no canonical total order on the circle, the extremal rules $\min / \max$ for phases should be understood as a modeling convention: we fix a reference direction (a branch cut) and interpret phases by their principal values in $[0, 2\pi)$, assuming that, for each hyperedge, the relevant phase data do not wrap around the cut (so that $\min / \max$ are meaningful).

If wrap-around effects are essential, replace $\min / \max$ in the phase part by a circular aggregation operator $\mathrm{Agg}_\circ$, e.g.,

$$\mathrm{Agg}_\circ^{\mathrm{mean}}(\{\theta_u\}) := \operatorname{Arg}\Big(\sum_u e^{i\theta_u}\Big), \qquad \mathrm{Agg}_\circ^{\mathrm{med}} \in \arg\min_{\theta \in [0, 2\pi)} \sum_u d(\theta, \theta_u),$$

where $d(\alpha, \beta) := \min_{k \in \mathbb{Z}} |\alpha - \beta + 2\pi k|$ is the angular distance. Then set, for instance, $\varphi_D(j) := \mathrm{Agg}_\circ(\{\varphi_{\alpha_j}(u) : u \in E_j\})$ and $\psi_D(j) := \mathrm{Agg}_\circ(\{\psi_{\alpha_j}(u) : u \in E_j\})$, and analogously define $\varphi_\eta(E), \psi_\eta(E)$ in Definition 2.8.

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2.4 Hypergraph with Complex q-Rung Orthopair Fuzzy Information

This subsection introduces the notion of a *complex q-rung orthopair fuzzy hypergraph*, obtained by equipping a hypergraph with the structure of a complex q-rung orthopair fuzzy set.

Definition 6(Complex q-Rung Orthopair Fuzzy Hypergraph). *(cf. [24]) Let Y be a (finite) nonempty set and let $q \geq 1$ be a fixed real number. A (finite)* complex q-rung orthopair fuzzy hypergraph *(Cq-ROFHG) is a (finite) ordered pair*

$$H = (Q, \eta),$$

where

$$Q = \{Q_1, Q_2, \ldots, Q_k\}$$

is a finite collection of complex q-rung orthopair fuzzy sets on Y. Each Q_j is given by

$$Q_j = \big\{(u, T_{Q_j}(u)e^{i\varphi_{Q_j}(u)}, F_{Q_j}(u)e^{i\psi_{Q_j}(u)}) : u \in Y\big\},$$

where, for every $u \in Y$,

$$T_{Q_j}(u), F_{Q_j}(u) \in [0,1], \quad \varphi_{Q_j}(u), \psi_{Q_j}(u) \in [0, 2\pi], \quad T_{Q_j}(u)^q + F_{Q_j}(u)^q \leq 1.$$

The mapping

$$\eta : \{E \subseteq Y : E \neq \emptyset\} \longrightarrow \big([0,1]^2 \times [0,2\pi]^2\big), \qquad \eta(E) = \big(T_\eta(E)e^{i\varphi_\eta(E)}, F_\eta(E)e^{i\psi_\eta(E)}\big),$$

assigns to each nonempty crisp hyperedge $E \subseteq Y$ a complex q-rung orthopair fuzzy pair. For every such hyperedge $E = \{a_1, \ldots, a_\ell\} \subseteq Y$, the following inequalities hold:

$$T_\eta(E) \leq \min_{1 \leq m \leq \ell} T_{Q_j}(a_m), \quad F_\eta(E) \leq \max_{1 \leq m \leq \ell} F_{Q_j}(a_m),$$

$$\varphi_\eta(E) \leq \min_{1 \leq m \leq \ell} \varphi_{Q_j}(a_m), \quad \psi_\eta(E) \geq \max_{1 \leq m \leq \ell} \psi_{Q_j}(a_m),$$

$$0 \leq T_\eta(E)^q + F_\eta(E)^q \leq 1.$$

Furthermore, the supports of the sets Q_j jointly cover the universe:

$$\bigcup_{j=1}^{k} \{u \in Y : T_{Q_j}(u) > 0 \text{ or } F_{Q_j}(u) > 0\} = Y.$$

3 Main Results of this paper

In this section, we develop and study two new SuperHyperGraph–based formalisms: the *Complex Intuitionistic Fuzzy SuperHypergraph* and the *Complex q-Rung Orthopair Fuzzy SuperHypergraph*. We supply precise definitions for each model and investigate their key structural properties.

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3.1 Complex Intuitionistic Fuzzy Information on SuperHypergraphs

A (finite) *complex intuitionistic fuzzy superhypergraph* is a layered superhypergraph in which each supervertex and each superedge is equipped with a *complex intuitionistic fuzzy label*: a membership value and a non-membership value, each represented by a magnitude in $[0,1]$ together with a phase angle in $[0,2\pi]$.

Definition 7(Complex intuitionistic fuzzy n-superhypergraph). *Let S be a finite nonempty base set and let $n \geq 0$ be fixed. For each $k \geq 0$, define the iterated powersets inductively by*

$$\mathrm{POS}^0(S) := S, \qquad \mathrm{POS}^{k+1}(S) := \mathrm{POS}\big(\mathrm{POS}^k(S)\big).$$

A (finite) complex intuitionistic fuzzy n-superhypergraph *on S is a tuple*

$$H = \big(V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E\big),$$

where:

- $V \subseteq \bigcup_{k=0}^{n} \mathrm{POS}^k(S)$ *is a nonempty* set *of (super)vertices. For convenience, one may regard V as layered via $V^{(k)} := V \cap \mathrm{POS}^k(S)$ $(0 \leq k \leq n)$.*
- $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$ *is a* set *of nonempty (super)edges; hence each $e \in E$ is a nonempty finite subset of V.*
- $T_V, F_V : V \to [0,1]$ *assign to each $v \in V$ its membership magnitude $T_V(v)$ and non-membership magnitude $F_V(v)$.*
- $\Xi_V, \Psi_V : V \to [0,2\pi]$ *assign to each $v \in V$ its membership phase $\Xi_V(v)$ and non-membership phase $\Psi_V(v)$.*
- $T_E, F_E : E \to [0,1]$ *assign to each $e \in E$ its membership magnitude $T_E(e)$ and non-membership magnitude $F_E(e)$.*
- $\Xi_E, \Psi_E : E \to [0,2\pi]$ *assign to each $e \in E$ its membership phase $\Xi_E(e)$ and non-membership phase $\Psi_E(e)$.*

These data are required to satisfy, for all $v \in V$ and $e \in E$,

$$T_V(v) + F_V(v) \leq 1, \qquad T_E(e) + F_E(e) \leq 1, \tag{4}$$

$$T_E(e) \leq \min_{x \in e} T_V(x), \qquad F_E(e) \leq \max_{x \in e} F_V(x), \tag{5}$$

$$\Xi_E(e) \leq \min_{x \in e} \Xi_V(x), \qquad \Psi_E(e) \geq \max_{x \in e} \Psi_V(x), \tag{6}$$

where, in the minima and maxima, each $e \in E$ is viewed as a subset of V.

Equivalently, the complex intuitionistic fuzzy pair

$$\Big(T_E(e)e^{i\Xi_E(e)},\ F_E(e)e^{i\Psi_E(e)}\Big)$$

assigned to a superedge e is bounded componentwise by the corresponding complex intuitionistic fuzzy values of its incident supervertices.

Remark(On phase aggregation and the use of $\min / \max$*).* In many applications, the phase components Ξ and Ψ represent cyclic effects such as *time-of-day*, *seasonality*, or *synchronization offsets*. Since phases live on the circle, there is no canonical total order on $[0,2\pi]$. Thus, the extremal rules in (6) should be understood as a *modeling convention*: we fix a reference direction and a branch cut, identify phases with their principal values in $[0,2\pi]$, and assume that the relevant phase data do not wrap around the cut (so that $\min / \max$ are meaningful).

If wrap-around effects are essential, one may replace $\min / \max$ in (6) by a circular aggregation operator, e.g. the circular mean

$$\mathrm{Arg}\Big(\sum_{x \in e} e^{i\Xi_V(x)}\Big),$$

or, more robustly, the Fréchet (circular) median with respect to the angular distance $d(\alpha,\beta) = \min_{k \in \mathbb{Z}} |\alpha - \beta + 2\pi k|$. All subsequent constructions remain valid after replacing (6) by the corresponding chosen circular aggregation rule.

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We next present concrete examples of complex intuitionistic fuzzy superhypergraphs.

Example 3(E-commerce fulfillment system as a complex intuitionistic fuzzy superhypergraph). Let

$$S = \{W_1, W_2, W_3\}, \qquad n = 1,$$

so that $\mathrm{POS}^1(S) = \mathrm{POS}(S)$. Construct two first-level supervertices (warehouse groups)

$$v_1 = \{W_1, W_2\}, \qquad v_2 = \{W_2, W_3\},$$

and set

$$V = \{v_1, v_2\} \subseteq \mathrm{POS}^1(S) = \mathrm{POS}(S).$$

Introduce a single superedge

$$e = \{v_1, v_2\}, \qquad E = \{e\} \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}.$$

Assign complex intuitionistic fuzzy labels to the supervertices by

$$\begin{aligned} T_V(v_1) &= 0.72, \quad \Xi_V(v_1) = \tfrac{\pi}{5}, \quad F_V(v_1) = 0.18, \quad \Psi_V(v_1) = \tfrac{3\pi}{5}, \\ T_V(v_2) &= 0.83, \quad \Xi_V(v_2) = \tfrac{\pi}{3}, \quad F_V(v_2) = 0.12, \quad \Psi_V(v_2) = \tfrac{\pi}{2}. \end{aligned}$$

For the superedge e, set

$$T_E(e) = 0.69, \qquad \Xi_E(e) = \tfrac{\pi}{5}, \qquad F_E(e) = 0.16, \qquad \Psi_E(e) = \tfrac{3\pi}{5}.$$

Then

$$T_V(v_1) + F_V(v_1) = 0.90 < 1, \qquad T_V(v_2) + F_V(v_2) = 0.95 < 1, \qquad T_E(e) + F_E(e) = 0.85 \leq 1.$$

Moreover, the incidence monotonicity constraints hold:

$$T_E(e) \leq \min\{0.72, 0.83\} = 0.72, \qquad F_E(e) \leq \max\{0.18, 0.12\} = 0.18,$$

$$\Xi_E(e) \leq \min\left\{\tfrac{\pi}{5}, \tfrac{\pi}{3}\right\} = \tfrac{\pi}{5}, \qquad \Psi_E(e) \geq \max\left\{\tfrac{3\pi}{5}, \tfrac{\pi}{2}\right\} = \tfrac{3\pi}{5}.$$

Consequently,

$$H = \big(V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E\big)$$

is a valid complex intuitionistic fuzzy 1-superhypergraph representing this fulfillment network. For reference, a schematic overview is given in Figure 3.

Below we record several basic results for complex intuitionistic fuzzy n-superhypergraphs.

Theorem 1(Reductions to standard models). *Let*

$$H = \big(V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E\big)$$

be a complex intuitionistic fuzzy n-superhypergraph on a base set S. Then the following special cases are obtained by imposing additional constraints.

1. ***Crisp n-superhypergraph.*** *If*

$$T_V \equiv 1, \quad F_V \equiv 0, \quad \Xi_V \equiv 0, \quad \Psi_V \equiv 0 \quad \text{on } V,$$

and

$$T_E \equiv 1, \quad F_E \equiv 0, \quad \Xi_E \equiv 0, \quad \Psi_E \equiv 0 \quad \text{on } E,$$

then all grades are trivial and H reduces to the underlying crisp n-superhypergraph

$$\mathrm{SuHyG}^{(n)} := (V, E).$$

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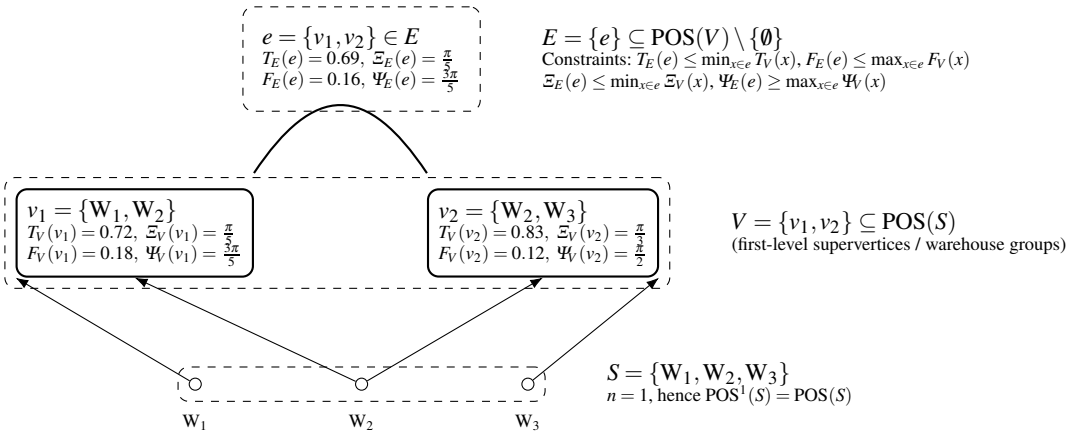
$e = \{v_1, v_2\} \in E$
$T_E(e) = 0.69,\ \Xi_E(e) = \frac{\pi}{5}$
$F_E(e) = 0.16,\ \Psi_E(e) = \frac{3\pi}{5}$
$E = \{e\} \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$
Constraints: $T_E(e) \le \min_{x\in e} T_V(x)$, $F_E(e) \le \max_{x\in e} F_V(x)$
$\Xi_E(e) \le \min_{x\in e} \Xi_V(x)$, $\Psi_E(e) \ge \max_{x\in e} \Psi_V(x)$
$v_1 = \{W_1, W_2\}$
$T_V(v_1) = 0.72,\ \Xi_V(v_1) = \frac{\pi}{5}$
$F_V(v_1) = 0.18,\ \Psi_V(v_1) = \frac{3\pi}{5}$
$v_2 = \{W_2, W_3\}$
$T_V(v_2) = 0.83,\ \Xi_V(v_2) = \frac{\pi}{2}$
$F_V(v_2) = 0.12,\ \Psi_V(v_2) = \frac{\pi}{2}$
$V = \{v_1, v_2\} \subseteq \mathrm{POS}(S)$
(first-level supervertices / warehouse groups)
$S = \{W_1, W_2, W_3\}$
$n = 1$, hence $\mathrm{POS}^1(S) = \mathrm{POS}(S)$
W_1
W_2
W_3

Fig. 3: E-commerce fulfillment example as a complex intuitionistic fuzzy 1-superhypergraph.

2.***Real-valued intuitionistic fuzzy n-superhypergraph.*** *If all phase functions vanish, i.e.*

$$\Xi_V \equiv \Psi_V \equiv 0 \quad \textit{and} \quad \Xi_E \equiv \Psi_E \equiv 0,$$

then the complex labels collapse to their magnitudes, and H becomes a real-valued intuitionistic fuzzy n-superhypergraph with vertex data (T_V, F_V) and edge data (T_E, F_E), satisfying

$$T_V(v) + F_V(v) \le 1 \quad (\forall v \in V), \qquad T_E(e) + F_E(e) \le 1 \quad (\forall e \in E),$$

together with the magnitude constraints (5).

3.***Complex intuitionistic fuzzy hypergraph*** *($n = 0$).* *If $n = 0$, then $\mathrm{POS}^0(S) = S$, hence $V \subseteq S$ and $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$. In this case no higher iterated powerset levels appear, and the data $(T_V, F_V, \Xi_V, \Psi_V)$ on V and $(T_E, F_E, \Xi_E, \Psi_E)$ on E satisfy exactly (4)–(6), i.e. H is precisely a complex intuitionistic fuzzy hypergraph on the base set S.*

Proof.(1) Under the stated assumptions, (4) becomes $1 + 0 \le 1$ on both vertices and edges. Moreover, for every $e \in E$,

$$T_E(e) = 1 = \min_{x\in e} 1, \qquad F_E(e) = 0 = \max_{x\in e} 0, \qquad \Xi_E(e) = 0 = \min_{x\in e} 0, \qquad \Psi_E(e) = 0 = \max_{x\in e} 0,$$

so (5)–(6) hold identically. Hence the structure is entirely determined by (V, E).

(2) If $\Xi_V \equiv \Psi_V \equiv 0$ and $\Xi_E \equiv \Psi_E \equiv 0$, then $T_V(v)e^{i\Xi_V(v)} = T_V(v)$ and $F_V(v)e^{i\Psi_V(v)} = F_V(v)$, and similarly for edges. The phase inequalities (6) become $0 \le 0$ and $0 \ge 0$, while (4) and (5) remain exactly the usual real-valued intuitionistic constraints.

(3) If $n = 0$, then $V \subseteq \mathrm{POS}^0(S) = S$ and $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$. The axioms (4)–(6) are exactly the defining conditions of a complex intuitionistic fuzzy hypergraph on S.

Theorem 2(Edge–vertex dominance (magnitudes and phases)). *Let H be a complex intuitionistic fuzzy n-superhypergraph. Then for every superedge $e \in E$,*

$$T_E(e) \le \min_{x\in e} T_V(x), \qquad F_E(e) \le \max_{x\in e} F_V(x),$$

$$\Xi_E(e) \le \min_{x\in e} \Xi_V(x), \qquad \Psi_E(e) \ge \max_{x\in e} \Psi_V(x).$$

Moreover, the membership pair $\big(T_E(e), \Xi_E(e)\big)$ is bounded coordinatewise by the incident vertex pairs $\big(T_V(x), \Xi_V(x)\big)$, while the non-membership pair $\big(F_E(e), \Psi_E(e)\big)$ is bounded in the mixed order F (usual $\le$) and Ψ (reverse $\ge$).

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*Proof.*These four inequalities are precisely the defining constraints (5)–(6). The final sentence is a restatement using the (phase-linearized) coordinatewise comparison.

Theorem 3(Induced sub-superhypergraph). *Let $V' \subseteq V$ be nonempty and define the induced edge set*

$$E' := \{\, e \in E : e \subseteq V' \,\}.$$

Let H' be obtained by restricting all vertex functions to V' and all edge functions to E', i.e.

$$H' = \big(V', E', T_V|_{V'}, F_V|_{V'}, \Xi_V|_{V'}, \Psi_V|_{V'}, T_E|_{E'}, F_E|_{E'}, \Xi_E|_{E'}, \Psi_E|_{E'}\big).$$

Then H' is again a complex intuitionistic fuzzy n-superhypergraph on the same base set S.

*Proof.*Since $V' \subseteq V$ and V satisfies the vertex-domain condition of an n-superhypergraph on S, the same domain condition holds for V'. If $e \in E'$, then $e \in E$ and $e \subseteq V'$, so $e \in \mathrm{POS}(V') \setminus \{\emptyset\}$; hence $E' \subseteq \mathrm{POS}(V') \setminus \{\emptyset\}$.

All constraints (4)–(6) are pointwise inequalities; therefore they remain true after restricting the relevant functions to V' and E'.

Theorem 4(Edge extension by unions). *Let $e_1, e_2 \in E$ (disjointness is not required). Define $e_3 := e_1 \cup e_2 \subseteq V$ and set*

$$T_E(e_3) := \min\{T_E(e_1), T_E(e_2)\}, \quad \Xi_E(e_3) := \min\{\Xi_E(e_1), \Xi_E(e_2)\},$$
$$F_E(e_3) := \max\{F_E(e_1), F_E(e_2)\}, \quad \Psi_E(e_3) := \max\{\Psi_E(e_1), \Psi_E(e_2)\}.$$

Let $T_E^+, F_E^+, \Xi_E^+, \Psi_E^+$ coincide with T_E, F_E, Ξ_E, Ψ_E on E, and take the above values on e_3. Then the extended structure

$$H^+ = \big(V, E \cup \{e_3\}, T_V, F_V, \Xi_V, \Psi_V, T_E^+, F_E^+, \Xi_E^+, \Psi_E^+\big)$$

is again a complex intuitionistic fuzzy n-superhypergraph on S.

*Proof.*Since $e_1, e_2 \subseteq V$, we have $e_3 = e_1 \cup e_2 \in \mathrm{POS}(V) \setminus \{\emptyset\}$, so $E \cup \{e_3\} \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$.

It remains to verify (4)–(6) for the new edge e_3.

Sum constraint. Let $T_i := T_E(e_i)$, $F_i := F_E(e_i)$ $(i = 1,2)$. Choose $j \in \{1,2\}$ such that $T_j = \min\{T_1, T_2\}$. Then $T_E(e_3) = T_j$, and $F_E(e_3) = \max\{F_1, F_2\} \le 1 - T_j$ because $\max\{F_1, F_2\} \le \max\{1 - T_1, 1 - T_2\} = 1 - \min\{T_1, T_2\} = 1 - T_j$. Hence $T_E(e_3) + F_E(e_3) \le 1$.

Magnitude constraints. By (5) applied to e_1, e_2,

$$T_E(e_i) \le \min_{x \in e_i} T_V(x), \qquad F_E(e_i) \le \max_{x \in e_i} F_V(x) \qquad (i = 1,2).$$

Therefore

$$T_E(e_3) = \min\{T_E(e_1), T_E(e_2)\} \le \min\Big\{\min_{x \in e_1} T_V(x), \min_{y \in e_2} T_V(y)\Big\} = \min_{z \in e_3} T_V(z),$$

and similarly

$$F_E(e_3) = \max\{F_E(e_1), F_E(e_2)\} \le \max\Big\{\max_{x \in e_1} F_V(x), \max_{y \in e_2} F_V(y)\Big\} = \max_{z \in e_3} F_V(z).$$

Phase constraints. By (6) applied to e_1, e_2,

$$\Xi_E(e_i) \le \min_{x \in e_i} \Xi_V(x), \qquad \Psi_E(e_i) \ge \max_{x \in e_i} \Psi_V(x) \qquad (i = 1,2).$$

Thus

$$\Xi_E(e_3) = \min\{\Xi_E(e_1), \Xi_E(e_2)\} \le \min_{z \in e_3} \Xi_V(z),$$

and

$$\Psi_E(e_3) = \max\{\Psi_E(e_1), \Psi_E(e_2)\} \ge \max_{z \in e_3} \Psi_V(z).$$

All axioms therefore hold for e_3, and the remaining edges are unchanged.

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Theorem 5(Level restriction). *Let H be a complex intuitionistic fuzzy n-superhypergraph on S, and fix m with $0 \le m \le n$. Define*

$$V_m := V \cap \mathrm{POS}^m(S), \qquad E_m := \{\, e \in E \mid e \subseteq V_m \,\}.$$

If $V_m \neq \emptyset$, then the restricted structure

$$H_m = \big(V_m, E_m, T_V|_{V_m}, F_V|_{V_m}, \Xi_V|_{V_m}, \Psi_V|_{V_m}, T_E|_{E_m}, F_E|_{E_m}, \Xi_E|_{E_m}, \Psi_E|_{E_m}\big)$$

is a complex intuitionistic fuzzy m-superhypergraph on S.

*Proof.*By construction $V_m \subseteq \mathrm{POS}^m(S)$, so the vertex-domain condition required for an m-superhypergraph holds for V_m. If $e \in E_m$, then $e \subseteq V_m$ and $e \neq \emptyset$, hence $e \in \mathrm{POS}(V_m) \setminus \{\emptyset\}$; thus $E_m \subseteq \mathrm{POS}(V_m) \setminus \{\emptyset\}$.

All axioms (4)–(6) are pointwise inequalities and therefore remain valid under restriction to V_m and E_m.

Corollary 1(Real-valued intuitionistic fuzzy specialization). *If $\Xi_V \equiv \Psi_V \equiv 0$ and $\Xi_E \equiv \Psi_E \equiv 0$, then H reduces to a real-valued intuitionistic fuzzy n-superhypergraph with data $\big(V, E, T_V, F_V, T_E, F_E\big)$.*

*Proof.*This is exactly Theorem 1 (2).

Corollary 2(Zero-order (hypergraph) specialization). *If $n = 0$, then H coincides with a complex intuitionistic fuzzy hypergraph on the base set S.*

*Proof.*This is exactly Theorem 1 (3).

3.2 Complex q-Rung Orthopair Fuzzy SuperHyperGraph

A (finite) complex q-rung orthopair fuzzy SuperHyperGraph is a hierarchical hypergraph whose supervertices and superedges are described by pairs of complex-valued membership and non-membership degrees; the corresponding real magnitudes are required to satisfy a q-rung orthopair condition.

Definition 8(Complex q-rung orthopair fuzzy n–SuperHyperGraph). *Let S be a nonempty set and let $n \ge 1$. Define the iterated powersets recursively by*

$$\mathrm{POS}^0(S) := S, \qquad \mathrm{POS}^{k+1}(S) := \mathrm{POS}\big(\mathrm{POS}^k(S)\big) \quad (k \ge 0).$$

Fix a real parameter $q \ge 1$. A complex q-rung orthopair fuzzy n–SuperHyperGraph *(*CqROF-nSHG*) on S is a* 10*-tuple*

$$H = \big(V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E\big),$$

where:

- $V \subseteq \mathrm{POS}^n(S)$ *is a nonempty set of* n-supervertices.
- $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$ *is a family of nonempty* n-superedges*; thus each $e \in E$ is a nonempty finite subset of V.*
- $T_V, F_V : V \to [0,1]$ *assign to every supervertex $v \in V$ its membership magnitude $T_V(v)$ and non-membership magnitude $F_V(v)$.*
- $\Xi_V, \Psi_V : V \to [0, 2\pi]$ *specify the membership and non-membership phase angles on supervertices.*
- $T_E, F_E : E \to [0,1]$ *and $\Xi_E, \Psi_E : E \to [0, 2\pi]$ provide the corresponding magnitudes and phases on superedges.*

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These maps are required to satisfy, for all $v \in V$ and all $e \in E$,

$$T_V(v)^q + F_V(v)^q \le 1, \qquad T_E(e)^q + F_E(e)^q \le 1, \tag{7}$$

$$T_E(e) \le \min_{x\in e} T_V(x), \qquad F_E(e) \le \max_{x\in e} F_V(x), \tag{8}$$

$$\Xi_E(e) \le \min_{x\in e} \Xi_V(x), \qquad \Psi_E(e) \ge \max_{x\in e} \Psi_V(x). \tag{9}$$

Equivalently, every superedge $e \in E$ is labeled by the complex orthopair

$$\big(T_E(e)e^{\mathrm{i}\Xi_E(e)}, F_E(e)e^{\mathrm{i}\Psi_E(e)}\big),$$

whose real magnitudes satisfy the q-rung condition (7), while its membership and non-membership components are bounded above or below by the extremal vertex values as in (8)–(9).

Example 4(Regional emergency care network as a complex 2-rung orthopair fuzzy 2–SuperHyperGraph). Let

$$S = \{h_1, h_2, h_3, h_4\}, \qquad n = 2,$$

be a set of four hospitals. We build a two-level clustering as follows:

$$\mathrm{POS}^1(S) = \big\{H_A = \{h_1, h_2\},\ H_B = \{h_3, h_4\}\big\},$$

where H_A and H_B represent two city-level hospital groups, and

$$\mathrm{POS}^2(S) = \big\{N_A = \{H_A\},\ N_B = \{H_B\}\big\},$$

where N_A and N_B denote two regional emergency–care networks. Set

$$V = \{N_A, N_B\} \subseteq \mathrm{POS}^2(S), \qquad E = \big\{\, e = \{N_A, N_B\}\big\} \subseteq \mathrm{POS}(V).$$

Fix $q = 2$. We interpret each supervertex N_A, N_B as a regional network, and the superedge e as a coordination corridor for sharing ambulances, specialists, and critical-care capacity between the two regions.

Assign to the supervertices the complex orthopair values

$$\begin{aligned} T_V(N_A) &= 0.80, \quad \Xi_V(N_A) = \tfrac{\pi}{4}, \quad F_V(N_A) = 0.30, \quad \Psi_V(N_A) = \tfrac{3\pi}{5}, \\ T_V(N_B) &= 0.75, \quad \Xi_V(N_B) = \tfrac{\pi}{3}, \quad F_V(N_B) = 0.35, \quad \Psi_V(N_B) = \tfrac{2\pi}{3}. \end{aligned}$$

For the superedge e, representing the inter-regional emergency coordination, define

$$T_E(e) = 0.70, \qquad \Xi_E(e) = \tfrac{\pi}{6}, \qquad F_E(e) = 0.32, \qquad \Psi_E(e) = \tfrac{3\pi}{4}.$$

We first verify the 2-rung orthopair conditions:

$$\begin{aligned} T_V(N_A)^2 + F_V(N_A)^2 &= 0.80^2 + 0.30^2 = 0.64 + 0.09 = 0.73 \le 1, \\ T_V(N_B)^2 + F_V(N_B)^2 &= 0.75^2 + 0.35^2 = 0.5625 + 0.1225 = 0.685 \le 1, \\ T_E(e)^2 + F_E(e)^2 &= 0.70^2 + 0.32^2 = 0.49 + 0.1024 = 0.5924 \le 1. \end{aligned}$$

Next, we check the magnitude bounds. Since

$$\min\{T_V(N_A), T_V(N_B)\} = \min\{0.80, 0.75\} = 0.75, \qquad \max\{F_V(N_A), F_V(N_B)\} = \max\{0.30, 0.35\} = 0.35,$$

we obtain

$$T_E(e) = 0.70 \le 0.75, \qquad F_E(e) = 0.32 \le 0.35,$$

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which is exactly the inequality system in (8).

For the phase bounds, we compute

$$\min\{\Xi_V(N_A),\Xi_V(N_B)\}=\min\left\{\tfrac{\pi}{4},\tfrac{\pi}{3}\right\}=\tfrac{\pi}{4},\qquad \max\{\Psi_V(N_A),\Psi_V(N_B)\}=\max\left\{\tfrac{3\pi}{5},\tfrac{2\pi}{3}\right\}=\tfrac{2\pi}{3},$$

so that

$$\Xi_E(e)=\tfrac{\pi}{6}\le\tfrac{\pi}{4},\qquad \Psi_E(e)=\tfrac{3\pi}{4}\ge\tfrac{2\pi}{3}.$$

Thus the phase constraints (9) are also satisfied.

All defining inequalities of a complex 2-rung orthopair fuzzy 2–SuperHyperGraph hold for the data above, so

$$H=\big(V,E,T_V,F_V,\Xi_V,\Psi_V,T_E,F_E,\Xi_E,\Psi_E\big)$$

is a valid complex 2-rung orthopair fuzzy 2–SuperHyperGraph modeling this two-level regional emergency care network. A schematic overview of this example is provided in Figure 4.

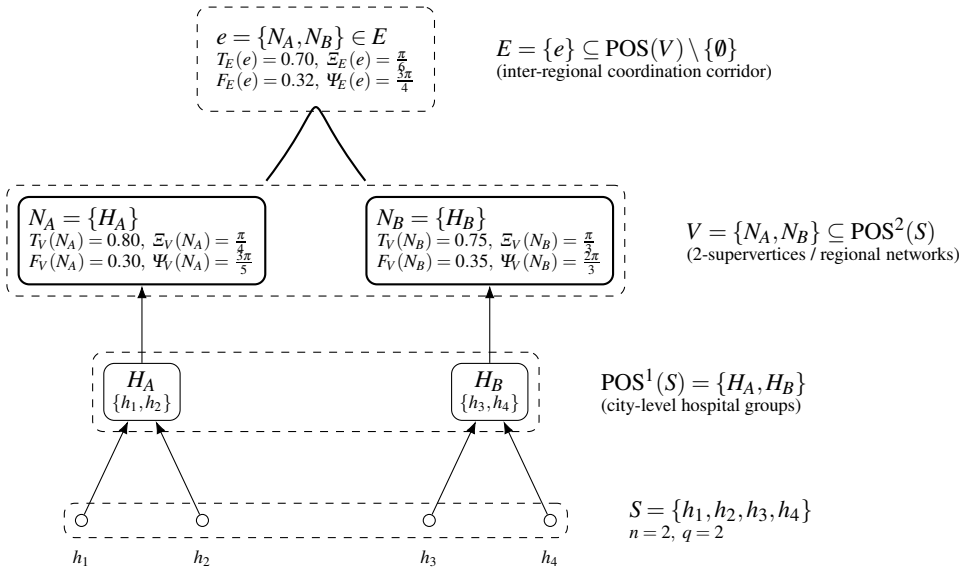

Fig. 4: Regional emergency care network as a complex 2-rung orthopair fuzzy 2-superhypergraph ($q=2$): hospitals $\to$ city-level groups $\to$ regional supervertices, connected by the superedge e.

Theorem 6(Reduction to classical and standard fuzzy frameworks). *Let H be a CqROF-nSHG as introduced in Definition 8. Then several well-known models arise as specializations of H:*

*1.*Crisp n–SuperHyperGraph. *Assume that, for every vertex $v\in V$ and every superedge $e\in E$,*

$$T_V(v)=1,\quad F_V(v)=0,\quad \Xi_V(v)=0,\quad \Psi_V(v)=0,$$

$$T_E(e)=1,\quad F_E(e)=0,\quad \Xi_E(e)=0,\quad \Psi_E(e)=0.$$

Under these choices all orthopair information becomes trivial, and H reduces to the underlying crisp n–SuperHyperGraph

$$\mathrm{SuHyG}^{(n)}:=(V,E).$$

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*2.*Fuzzy *n*–SuperHyperGraph. *Suppose in addition that*

$$\Xi_V \equiv \Psi_V \equiv 0, \qquad \Xi_E \equiv \Psi_E \equiv 0.$$

Then every complex orthopair is purely real. If we retain only the membership components, the structure

$$(V, E, T_V, T_E)$$

is a (real-valued) fuzzy n–SuperHyperGraph, where T_V and T_E give vertex and superedge membership degrees, respectively.

*3.*Intuitionistic fuzzy *n*–SuperHyperGraph. *If, moreover, $q = 1$ and still $\Xi_V \equiv \Psi_V \equiv 0$, $\Xi_E \equiv \Psi_E \equiv 0$, then the q-rung condition* (7) *becomes*

$$T_V(v) + F_V(v) \le 1, \qquad T_E(e) + F_E(e) \le 1,$$

for all $v \in V$ and $e \in E$. Together with the bounds (8)*, this yields a real intuitionistic fuzzy n–SuperHyperGraph*

$$(V, E, T_V, F_V, T_E, F_E).$$

*4.*Complex *q*-rung orthopair fuzzy hypergraph. *In the zero-order case $n = 0$ we have $\mathrm{POS}^0(S) = S$, so $V \subseteq S$ and $E \subseteq \mathrm{POS}(V)$. The families $(T_V, F_V, \Xi_V, \Psi_V)$ on V and $(T_E, F_E, \Xi_E, \Psi_E)$ on E then satisfy exactly the relations* (7)–(9) *with no higher powerset levels, which are precisely the axioms of a complex q-rung orthopair fuzzy hypergraph on the base set S.*

Proof.(1) Under the assignments in part (1) we obtain, for all $v \in V$ and $e \in E$,

$$T_V(v)^q + F_V(v)^q = 1^q + 0^q \le 1, \qquad T_E(e)^q + F_E(e)^q = 1^q + 0^q \le 1,$$

so (7) holds. Likewise, (8)–(9) simplify to

$$1 \le \min_{x \in e} 1 = 1, \quad 0 \le \max_{x \in e} 0 = 0, \quad 0 \le 0, \quad 0 \ge 0,$$

which are identities. Hence all graded information is degenerate and H is determined solely by (V, E), i.e. by the crisp *n*–SuperHyperGraph.

(2) If all phases vanish, each complex term $T_V(v)e^{\mathrm{i}\Xi_V(v)}$ (and similarly for the other components) coincides with its real magnitude. When we disregard the non-membership magnitudes and keep only T_V and T_E, the remaining inequalities inherited from (7)–(8) are exactly those defining a fuzzy *n*–SuperHyperGraph on (V, E).

(3) When $q = 1$, the orthopair condition reduces to

$$T_V(v) + F_V(v) \le 1, \qquad T_E(e) + F_E(e) \le 1.$$

Together with the magnitude bounds in (8), these are the standard axioms of a real intuitionistic fuzzy *n*–SuperHyperGraph with vertex pair (T_V, F_V) and edge pair (T_E, F_E).

(4) Finally, if $n = 0$, we have $\mathrm{POS}^0(S) = S$, whence $V \subseteq S$ and $E \subseteq \mathrm{POS}(V)$. The families $(T_V, F_V, \Xi_V, \Psi_V)$ and $(T_E, F_E, \Xi_E, \Psi_E)$, together satisfying (7)–(9), provide precisely the data of a complex *q*-rung orthopair fuzzy hypergraph on S. Therefore H belongs to this class in the zero-order case.

Theorem 7(Vertex–edge bounds). *For every superedge $e \in E$, the following inequalities hold:*

$$T_E(e) \le \min_{x \in e} T_V(x), \qquad F_E(e) \le \max_{x \in e} F_V(x),$$

$$\Xi_E(e) \le \min_{x \in e} \Xi_V(x), \qquad \Psi_E(e) \ge \max_{x \in e} \Psi_V(x).$$

Equivalently, the complex q-rung orthopair

$$\left(T_E(e)e^{\mathrm{i}\Xi_E(e)}, F_E(e)e^{\mathrm{i}\Psi_E(e)}\right)$$

attached to a superedge e is, coordinatewise, bounded by the extremal membership and non-membership values of its incident supervertices.

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*Proof.*By Definition 8, any CqROF-nSHG satisfies the constraints (8)–(9) for each superedge $e \in E$. The four displayed inequalities in the statement are exactly those constraints, written out explicitly, so the claim follows directly from the definition.

Theorem 8(Induced CqROF sub–SuperHyperGraph). *Let $V' \subseteq V$ be any nonempty subset and define*

$$E' = \{\, e \in E : e \subseteq V' \,\}.$$

Consider the restricted structure

$$H' = \big(V', E', T_V|_{V'}, \Xi_V|_{V'}, F_V|_{V'}, \Psi_V|_{V'}, T_E|_{E'}, \Xi_E|_{E'}, F_E|_{E'}, \Psi_E|_{E'}\big).$$

Then H' is again a complex q-rung orthopair fuzzy n–SuperHyperGraph on S.

*Proof.*Since $V \subseteq \mathrm{POS}^n(S)$ by Definition 8, every $v' \in V' \subseteq V$ also lies in $\mathrm{POS}^n(S)$. Hence V' is a valid vertex domain for an n–SuperHyperGraph. By construction each $e' \in E'$ satisfies $e' \subseteq V'$, so $E' \subseteq \mathrm{POS}(V') \setminus \{\emptyset\}$.

The functions

$$T_V, \Xi_V, F_V, \Psi_V,\ T_E, \Xi_E, F_E, \Psi_E$$

satisfy (7)–(9) on all of V and E. Since these conditions are imposed pointwise (for each vertex and each superedge), they continue to hold after restricting the domains to V' and E'. Thus all requirements of Definition 8 are met by

$$\big(V', E', T_V|_{V'}, \Xi_V|_{V'}, F_V|_{V'}, \Psi_V|_{V'}, T_E|_{E'}, \Xi_E|_{E'}, F_E|_{E'}, \Psi_E|_{E'}\big),$$

and H' is a CqROF-nSHG on S.

Theorem 9(Stability under edge union). *Let $e_1, e_2 \in E$ be disjoint superedges, i.e. $e_1 \cap e_2 = \emptyset$. Form their union*

$$e_3 := e_1 \cup e_2 \ \subseteq\ V,$$

and define

$$T_E(e_3) := \min\{T_E(e_1), T_E(e_2)\}, \quad \Xi_E(e_3) := \min\{\Xi_E(e_1), \Xi_E(e_2)\},$$
$$F_E(e_3) := \max\{F_E(e_1), F_E(e_2)\}, \quad \Psi_E(e_3) := \max\{\Psi_E(e_1), \Psi_E(e_2)\}.$$

Let $T_E^*, F_E^*, \Xi_E^*, \Psi_E^*$ coincide with T_E, F_E, Ξ_E, Ψ_E on all edges of E and take the above values on the new edge e_3. Then the extended structure

$$H^+ = \big(V,\ E \cup \{e_3\},\ T_V, \Xi_V, F_V, \Psi_V,\ T_E^*, \Xi_E^*, F_E^*, \Psi_E^*\big)$$

is again a complex q-rung orthopair fuzzy n–SuperHyperGraph.

*Proof.*Since $e_3 \subseteq V$, we have $e_3 \in \mathrm{POS}(V) \setminus \{\emptyset\}$, so the new edge set $E \cup \{e_3\}$ still lies in $\mathrm{POS}(V) \setminus \{\emptyset\}$.

(1) q-rung orthopair condition for e_3. Write

$$T_i := T_E(e_i), \qquad F_i := F_E(e_i) \qquad (i = 1, 2).$$

By (7) we know $T_i^q + F_i^q \le 1$ for $i = 1, 2$. Set

$$T_3 := \min\{T_1, T_2\}, \qquad F_3 := \max\{F_1, F_2\}.$$

Choose $k \in \{1, 2\}$ such that $F_3 = F_k$. Then $T_3 \le T_k$, hence $T_3^q \le T_k^q$, and we obtain

$$T_3^q + F_3^q \ \le\ T_k^q + F_k^q \ \le\ 1.$$

Thus (7) holds for the new edge e_3.

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(2) Magnitude constraints. From Theorem 7 applied to e_1 and e_2,

$$T_E(e_i) \le \min_{x\in e_i} T_V(x), \qquad F_E(e_i) \le \max_{x\in e_i} F_V(x), \quad i=1,2.$$

Therefore

$$\begin{aligned} T_E(e_3) &= \min\{T_E(e_1), T_E(e_2)\} \\ &\le \min\Big\{\min_{x\in e_1} T_V(x),\ \min_{y\in e_2} T_V(y)\Big\} = \min_{z\in e_3} T_V(z), \end{aligned}$$

and similarly

$$\begin{aligned} F_E(e_3) &= \max\{F_E(e_1), F_E(e_2)\} \\ &\le \max\Big\{\max_{x\in e_1} F_V(x),\ \max_{y\in e_2} F_V(y)\Big\} = \max_{z\in e_3} F_V(z). \end{aligned}$$

(3) Phase constraints. Again by Theorem 7,

$$\Xi_E(e_i) \le \min_{x\in e_i} \Xi_V(x), \qquad \Psi_E(e_i) \ge \max_{x\in e_i} \Psi_V(x), \quad i=1,2.$$

Hence

$$\Xi_E(e_3) = \min\{\Xi_E(e_1), \Xi_E(e_2)\} \le \min_{z\in e_3} \Xi_V(z),$$

and

$$\Psi_E(e_3) = \max\{\Psi_E(e_1), \Psi_E(e_2)\} \ge \max_{z\in e_3} \Psi_V(z).$$

All original edges still satisfy (7)–(9), and we have just verified these conditions for e_3 as well. Thus H^+ fulfills all axioms of Definition 8 and is a CqROF-nSHG on S.

Theorem 10(Restriction to a fixed level). *For any integer m with $1 \le m \le n$, set*

$$V_m := V \cap \mathrm{POS}^m(S), \qquad E_m := \{\, e \in E : e \subseteq V_m \},$$

and consider the restrictions of the ten structure maps,

$$T_V, F_V, \Xi_V, \Psi_V,\ T_E, F_E, \Xi_E, \Psi_E,$$

to V_m and E_m. Then

$$H_m = \big(V_m, E_m,\ T_V|_{V_m}, \Xi_V|_{V_m}, F_V|_{V_m}, \Psi_V|_{V_m},\ T_E|_{E_m}, \Xi_E|_{E_m}, F_E|_{E_m}, \Psi_E|_{E_m}\big)$$

is a complex q-rung orthopair fuzzy m–SuperHyperGraph on S.

Proof. By construction $V_m \subseteq \mathrm{POS}^m(S)$, so V_m is an admissible vertex set for an m–SuperHyperGraph. Every edge $e \in E_m$ satisfies $e \subseteq V_m$, hence $E_m \subseteq \mathrm{POS}(V_m) \setminus \{\emptyset\}$.

The inequalities (7)–(9) hold for all vertices in V and superedges in E by definition of a CqROF-nSHG. Since these conditions are pointwise, they remain true after restricting the maps to the smaller domains V_m and E_m. Consequently, the tuple H_m satisfies the same system of axioms with n replaced by m, and therefore is a CqROF-mSHG.

Theorem 11(Monotonicity with respect to q). *Let $T, F \in [0,1]$ and $1 \le q_1 \le q_2$. If*

$$T^{q_1} + F^{q_1} \le 1,$$

then also

$$T^{q_2} + F^{q_2} \le 1.$$

In particular, any complex q_1–rung orthopair fuzzy n–SuperHyperGraph automatically yields a complex q_2–rung orthopair fuzzy n–SuperHyperGraph for every $q_2 \ge q_1$, with the same underlying data.

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*Proof.*For fixed $t \in [0,1]$, the function $q \mapsto t^q$ is nonincreasing on $[1,\infty)$. Hence

$$T^{q_2} \le T^{q_1}, \qquad F^{q_2} \le F^{q_1},$$

and therefore

$$T^{q_2} + F^{q_2} \;\le\; T^{q_1} + F^{q_1} \;\le\; 1.$$

In a CqROF-nSHG, the only place where the parameter q appears is in the inequalities of the form $T^q + F^q \le 1$ (see (7)); the remaining relations (8)–(9) are independent of q. Thus, if the vertex and edge orthopairs satisfy the q_1–rung condition, they automatically satisfy the q_2–rung condition for all $q_2 \ge q_1$, so the same structure is valid for any larger parameter.

Theorem 12(Closure under intersection). *Let $H^{(1)}$ and $H^{(2)}$ be two CqROF-nSHGs on the same underlying pair (V,E). Define a new structure $H^{\cap}$ on (V,E) by taking, for each vertex $v \in V$,*

$$T_V^{\cap}(v) := \min\{T_V^{(1)}(v), T_V^{(2)}(v)\}, \quad F_V^{\cap}(v) := \max\{F_V^{(1)}(v), F_V^{(2)}(v)\},$$
$$\Xi_V^{\cap}(v) := \min\{\Xi_V^{(1)}(v), \Xi_V^{(2)}(v)\}, \quad \Psi_V^{\cap}(v) := \max\{\Psi_V^{(1)}(v), \Psi_V^{(2)}(v)\},$$

and, for each superedge $e \in E$,

$$T_E^{\cap}(e) := \min\{T_E^{(1)}(e), T_E^{(2)}(e)\}, \quad F_E^{\cap}(e) := \max\{F_E^{(1)}(e), F_E^{(2)}(e)\},$$
$$\Xi_E^{\cap}(e) := \min\{\Xi_E^{(1)}(e), \Xi_E^{(2)}(e)\}, \quad \Psi_E^{\cap}(e) := \max\{\Psi_E^{(1)}(e), \Psi_E^{(2)}(e)\}.$$

Then $H^{\cap}$ is again a CqROF-nSHG.

*Proof.*We first verify the q-rung condition for vertices; the edge case is analogous.

Fix $v \in V$ and write

$$T_i := T_V^{(i)}(v), \qquad F_i := F_V^{(i)}(v), \qquad i = 1,2,$$

so that $T_i^q + F_i^q \le 1$ by (7). Set

$$T^{\cap} := \min\{T_1, T_2\}, \qquad F^{\cap} := \max\{F_1, F_2\}.$$

Choose an index $k \in \{1,2\}$ such that $F^{\cap} = F_k$. Then $T^{\cap} \le T_k$, hence $(T^{\cap})^q \le T_k^q$, and therefore

$$(T^{\cap})^q + (F^{\cap})^q \;\le\; T_k^q + F_k^q \;\le\; 1.$$

Thus $T_V^{\cap}, F_V^{\cap}$ satisfy (7). The same argument applied to each edge $e \in E$ shows that $T_E^{\cap}, F_E^{\cap}$ also fulfill (7).

We now establish the magnitude and phase inclusions. Let $e \in E$ be fixed. From Theorem 7, applied to $H^{(1)}$ and $H^{(2)}$, we have

$$T_E^{(i)}(e) \le \min_{x \in e} T_V^{(i)}(x), \qquad F_E^{(i)}(e) \le \max_{x \in e} F_V^{(i)}(x), \quad i = 1,2.$$

Hence

$$\begin{aligned} T_E^{\cap}(e) &= \min\{T_E^{(1)}(e), T_E^{(2)}(e)\} \\ &\le \min\Big\{\min_{x \in e} T_V^{(1)}(x),\ \min_{x \in e} T_V^{(2)}(x)\Big\} \\ &= \min_{x \in e} \min\{T_V^{(1)}(x), T_V^{(2)}(x)\} \\ &= \min_{x \in e} T_V^{\cap}(x), \end{aligned}$$

and similarly

$$\begin{aligned} F_E^{\cap}(e) &= \max\{F_E^{(1)}(e), F_E^{(2)}(e)\} \\ &\le \max\Big\{\max_{x \in e} F_V^{(1)}(x),\ \max_{x \in e} F_V^{(2)}(x)\Big\} \\ &= \max_{x \in e} \max\{F_V^{(1)}(x), F_V^{(2)}(x)\} \\ &= \max_{x \in e} F_V^{\cap}(x). \end{aligned}$$

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An identical min–max rearrangement yields

$$\Xi_E^{\cap}(e) \le \min_{x\in e} \Xi_V^{\cap}(x), \qquad \Psi_E^{\cap}(e) \ge \max_{x\in e} \Psi_V^{\cap}(x).$$

Thus (8)–(9) hold for the intersection data, and $H^{\cap}$ satisfies all axioms of Definition 8.

Theorem 13(Union closure). *Let $H^{(1)}$ and $H^{(2)}$ be CqROF-nSHGs on the same underlying pair (V,E). Define a structure $H^{\cup}$ on (V,E) by, for each $v \in V$,*

$$T_V^{\cup}(v) := \max\{T_V^{(1)}(v), T_V^{(2)}(v)\}, \quad F_V^{\cup}(v) := \min\{F_V^{(1)}(v), F_V^{(2)}(v)\},$$
$$\Xi_V^{\cup}(v) := \min\{\Xi_V^{(1)}(v), \Xi_V^{(2)}(v)\}, \quad \Psi_V^{\cup}(v) := \max\{\Psi_V^{(1)}(v), \Psi_V^{(2)}(v)\},$$

and, for each $e \in E$,

$$T_E^{\cup}(e) := \max\{T_E^{(1)}(e), T_E^{(2)}(e)\}, \quad F_E^{\cup}(e) := \min\{F_E^{(1)}(e), F_E^{(2)}(e)\},$$
$$\Xi_E^{\cup}(e) := \min\{\Xi_E^{(1)}(e), \Xi_E^{(2)}(e)\}, \quad \Psi_E^{\cup}(e) := \max\{\Psi_E^{(1)}(e), \Psi_E^{(2)}(e)\}.$$

If these aggregated functions satisfy the inclusion bounds (8)–(9), then $H^{\cup}$ is a CqROF-nSHG.

Proof. We check the q-rung condition; the remaining inequalities are assumed to hold and therefore need not be derived.

Fix $v \in V$ and put

$$T_i := T_V^{(i)}(v), \qquad F_i := F_V^{(i)}(v), \qquad i = 1,2,$$

so $T_i^q + F_i^q \le 1$ by (7). Let

$$T^{\cup} := \max\{T_1, T_2\}, \qquad F^{\cup} := \min\{F_1, F_2\}.$$

Choose $k \in \{1,2\}$ with $T^{\cup} = T_k$. Then $F^{\cup} \le F_k$, so $(F^{\cup})^q \le F_k^q$, and we obtain

$$(T^{\cup})^q + (F^{\cup})^q \;\le\; T_k^q + F_k^q \;\le\; 1.$$

Hence $T_V^{\cup}, F_V^{\cup}$ satisfy (7). The same argument applied to each superedge $e \in E$ shows that $T_E^{\cup}, F_E^{\cup}$ also fulfill (7).

By hypothesis, the magnitude and phase relations (8)–(9) are satisfied by the maps $T_V^{\cup}, F_V^{\cup}, \Xi_V^{\cup}, \Psi_V^{\cup}$ and $T_E^{\cup}, F_E^{\cup}, \Xi_E^{\cup}, \Psi_E^{\cup}$. Consequently, $H^{\cup}$ meets all requirements of Definition 8 and is a CqROF-nSHG.

4 Algorithms

In this section, we design algorithms for the (finite) *Complex Intuitionistic Fuzzy SuperHyperGraph* and the (finite) *Complex q-Rung Orthopair Fuzzy SuperHyperGraph*, and we evaluate their correctness.

4.1 Complex Intuitionistic Fuzzy SuperHyperGraph

We next give a simple constructive procedure that, given vertex-level complex intuitionistic fuzzy data, produces edge-level data consistent with the defining constraints of a complex intuitionistic fuzzy n-superhypergraph.

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Algorithm 1: Construction of a Complex Intuitionistic Fuzzy n-SuperHyperGraph

Input: Finite base set S; integer $n \geq 0$;
nonempty vertex set $V \subseteq \bigcup_{k=0}^{n} \mathrm{POS}^k(S)$;
edge set $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$;
vertex maps $T_V, F_V : V \to [0,1]$ and $\Xi_V, \Psi_V : V \to [0,2\pi]$ such that $T_V(v) + F_V(v) \leq 1$ for all $v \in V$.
Output: A complex intuitionistic fuzzy n-superhypergraph
$H = (V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E)$.

```
// Step 0: input validation (optional)
foreach e ∈ E do
    if e = ∅ or e ⊈ V then
        report error and abort
foreach v ∈ V do
    if T_V(v) ∉ [0,1] or F_V(v) ∉ [0,1] or T_V(v) + F_V(v) > 1 then
        report error and abort
    if Ξ_V(v) ∉ [0,2π] or Ψ_V(v) ∉ [0,2π] then
        report error and abort

// Step 1: initialize edge-level maps
Initialize maps T_E, F_E : E → [0,1] and Ξ_E, Ψ_E : E → [0,2π]
foreach e ∈ E do
    T_E(e) ← 1
    F_E(e) ← 0
    Ξ_E(e) ← 2π
    Ψ_E(e) ← 0

// Step 2: propagate vertex data to edges by extremal aggregation
foreach e ∈ E do
    foreach x ∈ e do
        T_E(e) ← min{T_E(e), T_V(x)}
        F_E(e) ← max{F_E(e), F_V(x)}
        Ξ_E(e) ← min{Ξ_E(e), Ξ_V(x)}
        Ψ_E(e) ← max{Ψ_E(e), Ψ_V(x)}

return H = (V, E, T_V, F_V, Ξ_V, Ψ_V, T_E, F_E, Ξ_E, Ψ_E)
```

We next state and prove correctness and complexity guarantees for Algorithm 1.

Theorem 14(Correctness and time complexity of Algorithm 1). *Let S be finite, let $n \geq 0$, and let $V \subseteq \bigcup_{k=0}^{n} \mathrm{POS}^k(S)$ be nonempty and finite. Let $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$ be finite. Assume that the input maps $T_V, F_V : V \to [0,1]$ and $\Xi_V, \Psi_V : V \to [0,2\pi]$ satisfy $T_V(v) + F_V(v) \leq 1$ for all $v \in V$. Let T_E, F_E, Ξ_E, Ψ_E be produced by Algorithm 1. Then:*

1.(Correctness) *For every $e \in E$,*

$$T_E(e) = \min_{x \in e} T_V(x), \qquad F_E(e) = \max_{x \in e} F_V(x), \tag{10}$$

$$\Xi_E(e) = \min_{x \in e} \Xi_V(x), \qquad \Psi_E(e) = \max_{x \in e} \Psi_V(x). \tag{11}$$

In particular, the output $H = (V, E, T_V, F_V, \Xi_V, \Psi_V, T_E, F_E, \Xi_E, \Psi_E)$ satisfies the defining inequalities (4)–(6), hence is a complex intuitionistic fuzzy n-superhypergraph.

2.(Time complexity) *If $|V| = N$, $|E| = M$, and $\deg(e) := |e|$, then Algorithm 1 runs in time*

$$\mathcal{O}\Big(N + M + \sum_{e \in E} \deg(e)\Big),$$

i.e. linear in the size of the explicit incidence representation of (V, E).

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Proof.(1) Correctness. Fix $e \in E$ and write $e = \{x_1, \ldots, x_k\}$ with $k \geq 1$.

Membership magnitude. Algorithm 1 initializes $T_E(e) = 1$ and updates $T_E(e) \leftarrow \min\{T_E(e), T_V(x_j)\}$ for $j = 1, \ldots, k$. By induction on j,

$$T_E(e) \text{ after processing } x_1, \ldots, x_j \;=\; \min\{1, T_V(x_1), \ldots, T_V(x_j)\}.$$

After k steps, since $T_V(x_j) \in [0,1]$,

$$T_E(e) = \min\{T_V(x_1), \ldots, T_V(x_k)\} = \min_{x \in e} T_V(x).$$

This proves the first identity in (10).

Non-membership magnitude. Similarly, $F_E(e)$ is initialized to 0 and updated by $F_E(e) \leftarrow \max\{F_E(e), F_V(x_j)\}$. Hence

$$F_E(e) = \max\{F_V(x_1), \ldots, F_V(x_k)\} = \max_{x \in e} F_V(x),$$

giving the second identity in (10).

Phases. The same argument applies to phases: $\Xi_E(e)$ is initialized to 2π and updated by a minimum, while $\Psi_E(e)$ is initialized to 0 and updated by a maximum. Therefore (11) holds.

Verification of the axioms. The vertex constraints $T_V(v) + F_V(v) \leq 1$ are assumed. For edges, let $e \in E$ and choose $x^* \in e$ such that $F_V(x^*) = \max_{x \in e} F_V(x) = F_E(e)$. Then by (10), $T_E(e) = \min_{x \in e} T_V(x) \leq T_V(x^*)$. Hence

$$T_E(e) + F_E(e) \leq T_V(x^*) + F_V(x^*) \leq 1,$$

so (4) holds on edges. Finally, (5)–(6) follow directly from (10)–(11) (indeed they hold with equality for each e).

Thus the output is a complex intuitionistic fuzzy n-superhypergraph.

(2) Time complexity. The optional input validation loops take $\mathcal{O}(N+M)$ time. Initialization of the edge maps takes $\mathcal{O}(M)$ time. In the propagation step, each incidence $x \in e$ is visited exactly once and performs constant work. Therefore the total propagation cost is $\mathcal{O}\big(\sum_{e \in E} \deg(e)\big)$. Combining these yields

$$\mathcal{O}\Big(N + M + \sum_{e \in E} \deg(e)\Big),$$

as claimed.

4.2 Complex q-Rung Orthopair Fuzzy SuperHyperGraph

An illustrative algorithm for constructing a Complex q-Rung Orthopair Fuzzy SuperHyperGraph is presented in Algorithm 2.

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Algorithm 2: Construction of a Complex q-Rung Orthopair Fuzzy n–SuperHyperGraph

Input: Finite base set S; integers $n \geq 1$, $q \geq 1$;
vertex set $V \subseteq \mathrm{POS}^n(S)$;
edge family $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$;
for each $v \in V$, real numbers $T_V(v), F_V(v) \in [0,1]$ and phases $\Xi_V(v), \Psi_V(v) \in [0, 2\pi]$ such that $T_V(v)^q + F_V(v)^q \leq 1$.

Output: A Complex q-Rung Orthopair Fuzzy n–SuperHyperGraph
$H = (V, E, T_V, \Xi_V, F_V, \Psi_V, T_E, \Xi_E, F_E, \Psi_E)$.

```
// Step 0: basic input checks (nonemptiness, domain, q-rung condition)
```
foreach $e \in E$ **do**
  if $e = \emptyset$ *or* $e \not\subseteq V$ **then**
    report error and abort

foreach $v \in V$ **do**
  if $T_V(v) < 0$ *or* $T_V(v) > 1$ *or* $F_V(v) < 0$ *or* $F_V(v) > 1$ *or* $T_V(v)^q + F_V(v)^q > 1$ **then**
    report error and abort

```
// Step 1: initialize edge-level maps
```
Initialize maps T_E, F_E, Ξ_E, Ψ_E on E
foreach $e \in E$ **do**
  $T_E(e) \leftarrow 1$
  $F_E(e) \leftarrow 0$
  $\Xi_E(e) \leftarrow 2\pi$
  $\Psi_E(e) \leftarrow 0$

```
// Step 2: assign edge orthopairs using min/max aggregation
```
foreach $e \in E$ **do**
  foreach $x \in e$ **do**
    $T_E(e) \leftarrow \min\{T_E(e), T_V(x)\}$
    $F_E(e) \leftarrow \max\{F_E(e), F_V(x)\}$
    $\Xi_E(e) \leftarrow \min\{\Xi_E(e), \Xi_V(x)\}$
    $\Psi_E(e) \leftarrow \max\{\Psi_E(e), \Psi_V(x)\}$

```
// (Optional) Step 3: verify the q-rung condition for all edges
```
foreach $e \in E$ **do**
  if $T_E(e)^q + F_E(e)^q > 1$ **then**
    report error and abort

return $H = (V, E, T_V, \Xi_V, F_V, \Psi_V, T_E, \Xi_E, F_E, \Psi_E)$

The theorems concerning this algorithm are presented below.

Theorem 15(Correctness and time complexity of Algorithm 2). *Let S be a finite set, $n \geq 1$, $q \geq 1$, and let $V \subseteq \mathrm{POS}^n(S)$ and $E \subseteq \mathrm{POS}(V) \setminus \{\emptyset\}$ be finite. Assume that the input maps $T_V, F_V : V \to [0,1]$ and $\Xi_V, \Psi_V : V \to [0, 2\pi]$ satisfy*

$$T_V(v)^q + F_V(v)^q \leq 1 \quad \textit{for all } v \in V.$$

Then:

1. *The output H of Algorithm 2 is a Complex q-Rung Orthopair Fuzzy n–SuperHyperGraph; that is, for every $v \in V$ and $e \in E$,*

$$T_V(v)^q + F_V(v)^q \leq 1, \qquad T_E(e)^q + F_E(e)^q \leq 1,$$
$$T_E(e) \leq \min_{x \in e} T_V(x), \quad F_E(e) \leq \max_{x \in e} F_V(x),$$
$$\Xi_E(e) \leq \min_{x \in e} \Xi_V(x), \quad \Psi_E(e) \geq \max_{x \in e} \Psi_V(x).$$

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2.If $|V| = N$, $|E| = M$, *and* $\deg(e) = |e|$ *for each edge* $e \in E$, *then the running time of Algorithm 2 is*

$$\mathcal{O}\Big(N + M + \sum_{e \in E} \deg(e)\Big),$$

again linear in the size of the explicit representation of (V, E).

Proof.(1) *Correctness.* The vertex-level condition $T_V(v)^q + F_V(v)^q \leq 1$ is assumed as part of the input. We therefore focus on edges.

Fix an edge $e \in E$. As in the proof of Theorem 14, the iterative min–max updates produce

$$T_E(e) = \min_{x \in e} T_V(x), \qquad F_E(e) = \max_{x \in e} F_V(x),$$

$$\Xi_E(e) = \min_{x \in e} \Xi_V(x), \qquad \Psi_E(e) = \max_{x \in e} \Psi_V(x),$$

with $0 \leq T_E(e) \leq 1$ and $0 \leq F_E(e) \leq 1$.

We must now show the q-rung inequality $T_E(e)^q + F_E(e)^q \leq 1$. Let $x^* \in e$ be chosen so that

$$F_V(x^*) = \max_{x \in e} F_V(x) = F_E(e).$$

Then

$$T_E(e) = \min_{x \in e} T_V(x) \leq T_V(x^*).$$

Since $q \geq 1$ and both $T_E(e)$ and $T_V(x^*)$ lie in $[0,1]$, the function $t \mapsto t^q$ is nondecreasing on $[0,1]$, and hence

$$T_E(e)^q \leq T_V(x^*)^q.$$

By construction, $F_E(e) = F_V(x^*)$, so

$$T_E(e)^q + F_E(e)^q \leq T_V(x^*)^q + F_V(x^*)^q \leq 1,$$

where the last inequality uses the vertex q-rung condition for x^*. Thus every edge satisfies $T_E(e)^q + F_E(e)^q \leq 1$.

The magnitude and phase inclusion inequalities

$$T_E(e) \leq \min_{x \in e} T_V(x), \quad F_E(e) \leq \max_{x \in e} F_V(x),$$

$$\Xi_E(e) \leq \min_{x \in e} \Xi_V(x), \quad \Psi_E(e) \geq \max_{x \in e} \Psi_V(x)$$

follow immediately from the explicit formulas for T_E, F_E, Ξ_E, Ψ_E given above. Altogether, the data produced by the algorithm satisfy all axioms of a Complex q-Rung Orthopair Fuzzy n–SuperHyperGraph.

(2) *Time complexity.* The argument is identical in structure to that of Theorem 14. The initial checks scan all vertices and edges once and therefore require $\mathcal{O}(N+M)$ time. The nested loop over edges and incident vertices does a constant amount of work for each pair (e,x) with $x \in e$, so its running time is $\mathcal{O}\big(\sum_{e \in E} \deg(e)\big)$. Any final verification loop also has this order. Hence the total running time is

$$\mathcal{O}\Big(N + M + \sum_{e \in E} \deg(e)\Big),$$

which is linear in the size of the explicit incidence structure of (V, E). This completes the proof.

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5 Conclusion

In this research paper, we introduced two SuperHyperGraph-based frameworks: the (finite) *Complex Intuitionistic Fuzzy SuperHyperGraph* and the (finite) *Complex q-Rung Orthopair Fuzzy SuperHyperGraph*. These models lift complex intuitionistic fuzzy and complex q-rung orthopair fuzzy *hypergraph* structures to the SuperHyperGraph setting, providing a unified way to represent uncertainty together with nested, multi-level incidence patterns. In particular, the additional hierarchical layer enables one to encode both local interactions among elementary entities and higher-order interactions among *aggregated groups* (clusters, modules, regions, or components) within a single combinatorial object.

We also designed constructive algorithms for building these structures and analyzed their correctness and computational characteristics. From an applied viewpoint, these constructions suggest a concrete pipeline for decision-oriented modeling: (i) form hierarchical groupings from raw entities (e.g., items–categories–departments, hospitals–cities–regions, or facilities–clusters–networks), (ii) attach complex orthopair information to quantify reliability/uncertainty together with phase-like features (e.g., time-of-day effects, seasonality, periodic demand, or synchronization offsets), and (iii) propagate vertex-level information to superedges via extremal aggregation rules to obtain consistent higher-level interaction scores. This pipeline is relevant to decision science and operations research tasks in which decisions must be made under multi-criteria uncertainty across multiple organizational levels, such as hierarchical supply-chain risk assessment, multi-region healthcare coordination, resilient logistics planning, and reliability-aware service orchestration in cloud and communication networks.

As directions for future work, we plan to enrich these models by incorporating Neutrosophic Sets [27], Quadripartitioned Neutrosophic Sets [28], and Plithogenic Sets [29], yielding more flexible representations of indeterminacy and contradiction. We also intend to investigate analogous generalizations built on bidirected graphs [30], multidirected graphs [31], and semigraphs [32]. Finally, we will conduct computational experiments and case studies on real-world data (together with evaluations on benchmark datasets) to assess practical performance, parameter sensitivity, and the empirical usefulness of the proposed frameworks in downstream optimization and decision-support workflows.

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Data Availability The study is entirely theoretical in nature and does not make use of experimental, observational, or publicly archived datasets. All statements, definitions, and results are self-contained within the text of this manuscript.
Ethical Approval Because the research does not involve experiments on humans, animals, or sensitive personal data, no formal ethics committee approval was required.
Use of Generative AI and AI-Assisted Tools We use generative AI and AI-assisted tools for limited purposes such as English grammar and style checking. These tools are not used in any manner that compromises academic integrity or violates ethical standards.
Conflicts of Interest The authors declare that they have no financial, professional, or personal relationships that could be perceived as affecting the analysis, findings, or conclusions reported in this manuscript.

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